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Flat bands and compact localized states in magnonic crystals

Grzegorz Centała[1], Jarosław W. Kłos[1], *Member, IEEE*
[1]Institute of Spintronics and Quantum Information, Faculty of Physics,
Adam Mickiewicz University, Poznań, klos@amu.edu.pl

Lieb lattice is one of the simplest bipartite lattices where flat bands and compact localized states are observed. We demonstrate the possibility of realizing a magnonic Lieb lattice in a planar structure of sub-micron in-plane sizes. The Ga-doped YIG layer with cylindrical inclusions (without Ga content) arranged in a Lieb lattice has been investigated numerically (finite element method). Such a design reproduces the Lieb lattice of nodes (inclusions) coupled to each other by the matrix with the compact localized states in flat bands.

***Index Terms*—magnonic crystals, flat bands, compact localized states**

I. Introduction

THE LIEB LATTICE is a complex lattice, where the nodes of minority (square) sublattice, connect to each other only via the nodes from other two majority (square) sublattices (Fig. 1). In such system the flat bands are observed in the absence of defects [1]. An intuitive explanation for the presence of the flat bands is the internal isolation of excitations located in one of the sublattices. The canceling of excitations at one sublattice is the result of destructive interference and local symmetry within the complex unit cell [2]. When only one of the sublattices is excited, the other sublattice does not mediate the coupling between neighboring unit cells, and the phase difference between the cells is irrelevant to the energy (or the frequency) of the eigenmode on the whole lattice - i.e. the Bloch function. The Bloch functions for flat band are then degenerated for every value of wave number $\mathbf{k}$. The linear combination of Bloch functions differing in $\mathbf{k}$ (with a coefficients $f(\mathbf{k})e^{\mathbf{R}\cdot\mathbf{k}}$, where $f(\mathbf{k})$ is arbitrary continuous function) are localized around lattice vector $\mathbf{R}$, similarly like Wannier functions – these states are called compact localized states.

The topic of Lieb lattices and other periodic structures with compact localization and flat bands was renewed [1] about 10 years ago when physical realizations of synthetic Lieb lattices began to be considered for electronic systems, optical lattices, superconducting systems, in phononics, and photonics. Lieb lattices have also been studied in the context of magnetic properties, mainly due to the possibility of enhancing ferromagnetism in systems of correlated electrons [3], where the occurrence of flat bands with zero kinetic energy was used to expose the interactions. However, the comprehensive studies of spin waves in nanostructures that realize magnonic Lieb lattices and focus on wave effects in a continuous model have not been carried out so far. This digest briefly summarizes the results presented in our paper [4], where we proposed the realization of the magnonic Lieb lattice.

II. Magnonic Lieb lattice

We considered the planar magnonic crystal (MC) – see Fig. 1, because of its relative ease of fabrication and experimental characterization. We propose realistic systems that

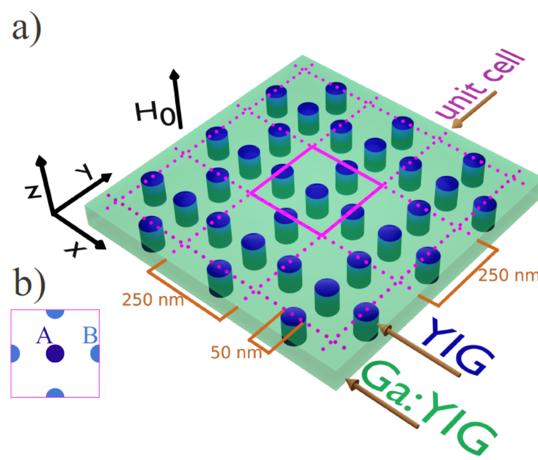

Fig. 1. (a) The magnonic Lieb lattice has a form of planar magnonic crystal consisting of YIG cylindrical nanoelements embedded within Ga:YIG matrix. Dimensions of the ferromagnetic unit cell for the basic Lieb lattice are equal to 250x250x59 nm. (b) The unit cell contains three inclusions of 50 nm in diameter. The separation between centers of inclusions is equal to 125 nm.

mimic the main features of the tight-binding model of Lieb lattice [5]. Investigated MCs consist of yttrium-iron-garnet (YIG) doped with gallium (Ga:YIG) matrix and YIG cylindrical inclusions arranged in Lieb lattice (Fig. 1(b)). Doping YIG with Gallium is a procedure where magnetic Fe^{3+} ions are replaced by non-magnetic Ga^{3+} ions. This method not only decreases saturation magnetization M_S but, simultaneously, arises uniaxial out-of-plane anisotropy, that ensures the out-of-plane orientation of static magnetization in Ga:YIG layer at a relatively low external field applied perpendicularly to the layer. Discussed geometry, i.e. forward volume magnetostatic spin-wave configuration, does not introduce an additional anisotropy in the propagation of spin waves related to the orientation of static magnetization. The design of the Lieb lattice requires the partial localization of spin-wave in inclusions, which can be treated as an approximation of the nodes from the tight-binding model. The condition which guarantees the focusing of magnetization dynamics inside the inclusions is fulfilled in the frequency range below the ferromagnetic resonance (FMR) frequency of the out-of-

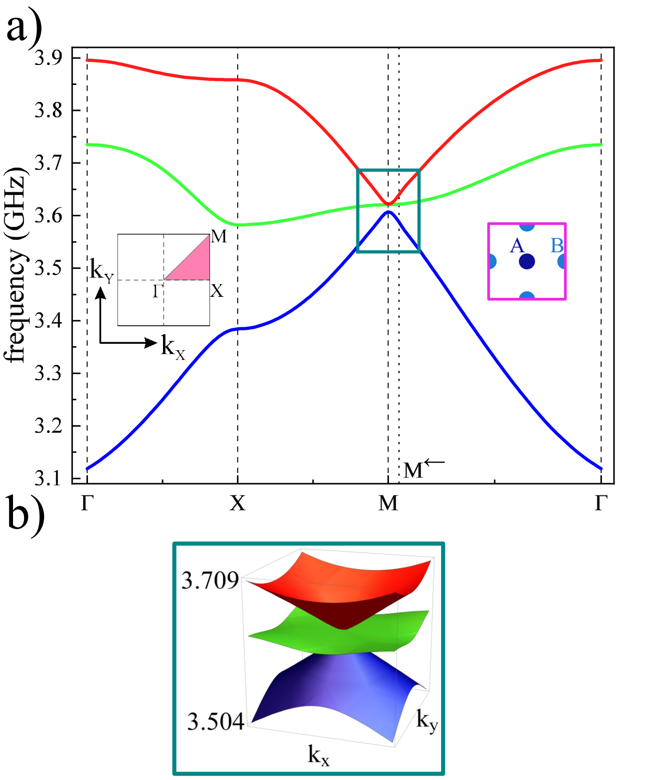

Fig. 2. Dispersion relation for the basic magnonic Lieb lattice containing three inclusions in the unit cell: one inclusion A from minority sublattice and two inclusions B from majority sublattices (see Fig. 1) (a) The dispersion relation is plotted along the high-symmetry path Γ-X-M-Γ (see the inset). The lowest band (blue) and the highest band (red) create Dirac cones almost touching (b) in the M point. The middle band (green) is relatively flat in the vicinity of the M point.

plane magnetized layer made of Ga:YIG (matrix material): $f_{\mathrm{FMR,Ga:YIG}} = 4.95$ GHz and above the FMR frequency of out-of-plane magnetized layer made of YIG (inclusions material): $f_{\mathrm{FMR,YIG}} = 2.42$ GHz. These limiting values were obtained using the Kittel formula for out-of-plane magnetized film: $f_{\mathrm{FMR}} = (\gamma/2\pi)|\mu_0 H_0 + \mu_0 H_{\mathrm{ani}} - \mu_0 M_S|$, where we used the following values of material parameters [6] for YIG: gyromagnetic ratio $\gamma = 177$ rad $\mathrm{T}^{-1}\mathrm{ns}^{-1}$, magnetization saturation $\mu_0 M_S = 182.4$ mT, exchange stiffness constant $A = 3.68$ pJ m^{-1}, (first-order) uniaxial anisotropy field $\mu_0 H_{\mathrm{ani}} = -3.5$ mT, and for Ga:YiG: $\gamma = 179$ rad $\mathrm{T}^{-1}\mathrm{ns}^{-1}$, $\mu_0 M_S = 20.2$ mT, $A = 1.37$ pJ m^{-1}, $\mu_0 H_{\mathrm{ani}} = 94.1$ mT. Due to the presence of out-of-plane anisotropy and relatively low saturation magnetization, we could consider a small external magnetic field $\mu_0 H_0 = 100$ mT to reach saturation state.

III. Method and Results

We used the COMSOL Multiphysics to implement the Landau-Lifshitz-Gilbert (LLG) equation and performed finite element method computation for the defined geometry of magnonic Lieb lattices. The frequency spectrum of eigenmodes depending on wave vector was determined by solving linearized LLG equations directly in the frequency domain. The results are shown in Fig. 2.

Three lowest bands form a band structure that is similar to the dispersion relation known from the tight-binding model [7]. The first and third band form Dirac cones at M point, separated by a tiny gap of about 15 MHz. The possible mechanism responsible for opening the gap is a small difference in the demagnetizing field in the areas of inclusions A (from the minority lattice) and inclusions B (from two majority sublattices). Inclusions A (B) have four (two) neighbors of type B (A). Although inclusions A and B have the same size and are made of the same material, the static field of demagnetization inside them differs slightly due to the different vicinity. This effect is equivalent to the dimerization of the Lieb lattice by varying the energy of the nodes in the tight-binding model, which leads to the opening of a gap between Dirac cones and parabolic flattening of them in very close proximity to the M point.

We proved that the second band supports the compact localized states regardless of its finite width [4]. The phases and amplitudes of the spin waves concentrated in inclusion A and two inclusions B are in agreement with the predictions of tight-binding models [8]:

$$|m_{\mathbf{k}}\rangle = \Big[\underbrace{-\cos(k_y a/2)}_{B_x}, \underbrace{0}_{A}, \underbrace{\cos(k_x a/2)}_{B_y}\Big]^{\mathrm{T}}, \qquad (1)$$

where $\mathbf{k} = [k_x, k_y]$ is wave vector and a size of unit cell (Fig. 1(b)).

IV. Conclusions

The magnonic Lieb lattices allow considering many problems related to dynamics, localization, and interactions in flat-band systems taking the advantage of the magnonic systems: presence and possibility of tailoring of long-range interactions, intrinsic non-linearity, etc.

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